RAMAKRISHNA MISSION VIDYAMANDIRA
(Residential Autonomous College affiliated to University of Calcutta)
B.Sc. FOURTH SEMESTER TAKE-HOME TEST/ASSIGNMENT, AUGUST 2021
SECOND YEAR [BATCH 2019-22]

Date : 07/08/2021 **MATHEMATICS**
Time : 11am – 1pm **Paper MACT 8** Full Marks : 50

Instructions to the Candidates

- Write your **College Roll No, Year, Subject & Paper Number** on the top of the **Answer Script**.
- Write your **Name, College Roll No, Year, Subject & Paper Number** on the **text box of your e-mail**.
- Read the instructions given at the beginning of each group/unit carefully.
- Only handwritten (by blue/black pen) answer-scripts will be permitted.
- Try to answer all the questions of a single group/unit at the same place.
- All the pages of your answer scripts must be numbered serially by hand.
- In the last page of your answer-scripts, please mention the total number of pages written so that we can verify it with that of the scanned copy of the scripts sent by you.
- For an easy scanning of the answer scripts and also for getting better image, students are advised to write the answers in single side and they must give a minimum 1 inch margin at the left side of each paper.
- After the completion of the exam, scan the entire answer script by using Clear Scan: Indy Mobile App OR any other Scanner device and make a **single PDF file (Named as your College Roll No)** and send it to [redacted]

Group A : Metric Spaces

Answer as many questions you can. Maximum you can obtain is 30 marks in group A.

1. (a) Define a metric 'd' on $\mathbb{Q}$ such that each point of $(\mathbb{Q}, d)$ is isolated. Give explanation. [4]
   (b) Suppose A, B are closed in $\mathbb{R}$. Is $A + B = \{x + y : x \in A, y \in B\}$ closed in $\mathbb{R}$? Justify. [3]
   (c) Suppose A, B are closed in $\mathbb{R}$ such that $A + B \subseteq [0, \infty)$. Is $A + B$ closed in $\mathbb{R}$? Justify. [5]
2. (a) "Every bounded sequence has a convergent subsequence " - Is it true in a metric space? Justify your answer. [3]
   (b) Let G be open in $\mathbb{R}$ such that $0 \notin G$. Prove that $gG = \{gx : x \in G\}$ is open in $\mathbb{R}, \forall g \in G$. [3]
3. (a) Find a sequence $\{D_n\}$ of subsets of $\mathbb{R}$ such that each D_n is countable and dense in $\mathbb{R}$ but $\bigcap_{n=1}^{\infty} D_n$ is not dense in $\mathbb{R}$. [2]
   (b) Suppose 'd' is the discrete metric on $\mathbb{R}$. Is $(\mathbb{R}, d)$ second countable? Justify. [2]
4. Prove that the space l_∞ is not separable. Is l_∞ totally bounded? Give justification. [5+1]
5. (a) Let

$$\begin{aligned} A &= \{(x, y) \in \mathbb{R}^2 : x \notin \mathbb{Q} \text{ or } y \notin \mathbb{Q}\} \\ B &= \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, y \in \mathbb{N}\} \text{ and} \\ C &= \{(x, y) \in \mathbb{R}^2 : x^2 \leq 4, 2 \leq y^2 \leq 4\}. \end{aligned}$$

   Justify whether A, B, C are compact in $\mathbb{R}^2$. [3]
   (b) Show that a connected metric space with atleast two distinct points is uncountable. [3]
   (c) Does there exist a set in $\mathbb{R}$ such that BdA is connected? [2]

Group B : Abstract Algebra III

Answer as many questions you can. Maximum you can obtain is 20 marks in group B.

6. (a) Which of the following groups can be written as direct product of proper subgroups and why? [6]

   i. $\mathbb{Z}_9$, ii. $\mathbb{Z}_{21}$, iii. D_4.

   (b) Find the number of elements of order 3 in $\mathbb{Z}_9 \times \mathbb{Z}_3$. [4]

   (c) What are the elements of finite order in $\mathbb{Z}_4 \times \mathbb{Z}$? Justify. [3]

7. (a) Suppose G is a nonabelian group of order 8. Find the order of $Z(G)$. [3]

   (b) If G is a group with exactly three subgroups then show that $o(G) = p^2$ for some prime p. [3]

   (c) Prove that a group of order 90 is not simple. [5]

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